COMPOSITE TIMBER-CONCRETE STRUCTURES

L. Surovec¹, M. Slivansky²

¹, ² Department of Steel and Timber Structures, Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11, 81005 Bratislava, Slovakia

e-mail: lukas.surovec@stuba.sk

Abstract

Paper deals with design of composite timber-concrete beams. Nowadays, timber-concrete beams are designed according to Eurocode 5, Annex B. This computational technique, also called γ-method, uses coefficient γ to consider compliance of shear connection at the material interface. Regarding linear elastic properties and slip between timber and concrete, tensional stress at the bottom surface of the concrete can be obtained in some cases. In respect of almost no tensional strength of concrete it is suitable to reinforce this layer with steel bars. A way how to exclude tensional stress from calculation is mentioned. Alternatively to a standard calculation, finite element method (FEM) as computational tool can be used. Approach how to create FEM model is described.

Keywords composite structures, timber, concrete

1. INTRODUCTION

Composite timber-concrete structures are mainly represented by composite floors. These floors utilize benefits of both materials, whether mechanical or physical. In opposition to classical timber beams using composite technology these structures have higher resistance and also more favourable values of deformations and vibration are reached. In civil engineering we can meet with timber-concrete in the structures of road bridges with short or middle span and also in the structures of foot or cycle bridges (fig. 1).

There are many possibilities how to create shear connection between timber and concrete. The most popular are dowel type fasteners – nails or screws with orientation perpendicular to wood grains or under 45° to wood grains. Also grooved connections, connection with perforated steel plates or connection with glued-in bars are used very often.

Fig.1: Composite timber-concrete foot bridge in Schwäbisch Gmünd, Germany [8].
2. DESIGN ACCORDING TO EN 1995

With the slip at the interface between timber and concrete we can evaluate the shear connection between these two materials as semi-rigid. This facts needs to be considered during the calculation. When we design composite timber-concrete floor we can take into account effective “T” section (fig. 2). Calculation approach is elected in technical standard EN 1995-1-1/Annex B. This method, also known as γ-method, express rate of shear connection using coefficient γ as follows:

\[ \gamma_2 = 1 \]

\[ \gamma_1 = \left[ 1 + \frac{\pi^2 E_i A_i s}{KL^2} \right]^{-1} \]

where:
- \( \gamma_i \) - coefficient \( \gamma \) of \( i^{th} \) part of cross section (see fig. 2),
- \( E_i \) - Young’s modulus of elasticity of \( i^{th} \) part of cross section,
- \( A_i \) - area of \( i^{th} \) part of cross section,
- \( s \) - spacing of fasteners longitudinal with axis of beam,
- \( K \) - slip modulus of fasteners (\( K_{ser} \) for service limit state – SLS, \( K_u \) for ultimate limit state – ULS),
- \( L \) - span of the beam.

Fig.2: Calculation scheme according to EN 1995-1-1/Annex B

If timber and concrete have no shear connection, coefficient \( \gamma_1 \) has value 0. If they are totally rigidly connected, coefficient \( \gamma_1 \) has value 1. It means that for all real cases the value of coefficient \( \gamma_1 \) lies between 0 and 1.

When we are able to determine coefficient \( \gamma_i \), we can continue with calculation of effective bending stiffness:

\[ (EI)_{eff} = \sum_{i=1}^{n} \left( E_i I_i + \gamma_i E_i A_i a_i^2 \right) \]

where:
- \( I_i \) - moment of inertia of \( i^{th} \) part of cross section,
- \( a_i \) - distance from centroid of \( i^{th} \) part of cross section to centroid of whole cross section.

Distances \( a_i \) are defined by following formulas:
Subsequently it is possible to calculate corresponding normal stresses:

\[ \sigma_i = \frac{\gamma_i E_i a_i M}{(EI)_{\text{eff}}} \]  \quad \text{(6)}

\[ \sigma_{m,i} = \frac{0.5 E_i h_i M}{(EI)_{\text{eff}}} \]  \quad \text{(7)}

and shear stress in the timber:

\[ \tau_{2,\text{max}} = \frac{0.5 E_z h^2}{(EI)_{\text{eff}}} V \]  \quad \text{(8)}

Finally the force acting on a one fastener or a group of fasteners set side by side is expressed by expression:

\[ F_i = \frac{\gamma_1 E_i A_i a_i s_i}{(EI)_{\text{eff}}} V \]  \quad \text{(9)}

This calculation method has some limits. It is possible to use it for single span beams. Formulas are also valid for continuous beams and cantilevers but real span width \( L \) must be replaced by the value of \( 4/5L \) for continuous beams and \( 2L \) for cantilevers. Other assumptions are:

- the individual parts (timber and concrete) must be connected with mechanical fasteners with slip modulus \( K \),
- spacing between the fasteners is either constant \((s_i = s_i)\) or varies uniformly by the shear force by \( s_{\text{min}} \) to \( s_{\text{max}} \), while \( s_{\text{max}} \leq 4s_{\text{min}} \),
- loading acts in \( z\)-direction and giving a bending moment \( M=M(x) \) varying sinusoidally or parabolically and a shear force \( V=V(x) \).

Besides the verification with short-term material properties, verification that considers creep of concrete, timber and connection, shrinkage of concrete and other long-term behaviour of structure must be evaluated [1].

3. CALCULATION CONSIDERING EFFECTIVE HEIGHT OF CONCRETE

Chapter 2 contains technical standard procedure for calculation of composite timber-concrete beams. This method allows formation of tensional stresses in concrete part of cross section. In fact when some level of tensional stress in concrete is reached a crack occur and it will spread up to the neutral axis. It leads to redistribution of bending stiffness on beam. To consider this phenomenon it is possible to modify calculation model in the sense that only effective compressed height of concrete will be taken into account (fig. 3). Assumption is that \( \sigma_j = \sigma_{m,i} \). After substitution of \( x \) for \( h_j \) to the equation (7) and consequent comparation of expressions (6) and (7) we get:

\[ \gamma_i a_i = 0.5 x \]  \quad \text{(10)}
Establishing expression (11) to the equation (4) and consequential modification we get quadratic equation (12). During the derivation of (12) all cross section properties of concrete have to be related to the centroid of effective compressed height of concrete. We are able to solve this equation exactly, e.g. using discriminant. Modification that is able to consider tensional reinforcement of concrete is very similar. For more information about this method see [2].

\[ a_1^2 \left( 4\gamma_1^2 E_1 h_1 \right) + a_1 \left[ 2E_2 A_2 (1 + \gamma_1) - E_2 A_2 (2h_1 + 2t_d + h_2) \right] = 0 \]  \hspace{1cm} ((12))

Fig.3: Modified calculation scheme considering effective compressed height of concrete

4. MODELLING OF COMPOSITE TIMBER-CONCRETE BEAMS

Behaviour of structure at interface between timber and concrete is influenced by many factors. Consideration of compliance of connection of these two components is characterized by slip modulus. It says how big force should be applied on the connection to get slip of value 1 mm. Values of this characteristic are for some simple fasteners (nails or screws perpendicular to wood grains) mentioned in technical standard for designing of timber structures – EN 1995-1-1 [1]. For connection that uses screws oriented in 45° to wood grains are these values stated in technical documentation of specific producer. Other opportunity is to make experimental push-out tests or make numerical model of specific connection.

Numerical models can be also used for global analysis of composite timber-concrete beams. Standard software for mechanical analysis that uses finite element method (FEM) often contains library of composite cross section but timber-concrete beams are very specific. As mentioned in previous chapter, shear connection of these two materials is not totally rigid but it is not a problem to create compliant connection using standard tools. For analysis in 2D we are able to make model of timber and concrete with beam elements of rectangular cross section. Concrete beam lies above timber beam in distance that is equal to distance of centroids of these two components. Supports can be applied on timber beam, load on concrete beam. Next we have to create the interface between these two materials. To transfer vertical load it is possible to use vertical rigid links that transfer only axial forces. To transfer horizontal force caused by shear flow we can use horizontal beams with axial stiffness equal to slip modulus of fastener. These beams are located on horizontal axis at the interface between timber and concrete. Spacing of these beams is equal to spacing of fasteners. Very important thing is that these beams have to transfer only axial load. For connection of these beams to timber and concrete is possible to use rigid links. Detail of this model is showed in figure 4. For 3D analysis it is possible to replace concrete beam for shell elements with adequate height.
4.1 Comparison of global analysis using FEM software and using standard calculation

To verify modelling procedure mentioned above the comparative study was evaluated. Three calculation models were compared – technical standard calculation (Model EC), FEM calculation with timber and concrete beam elements (Model Beam) and FEM calculation with timber as beam elements and concrete as shell elements (Model Shell). Results were compared on simple supported beam with span of 5.4m. Cross section is made by timber beam with 100mm width and 200mm height, concrete slab with height of 80mm and effective width of 800mm. Slip modulus was varying from 0 to 50000N/mm. For a better notion about behaviour of composite beam also totally rigid connection was considered. Chart of deflection depending on slip modulus is shown on figure 5.

4.2 Contribution of the transverse stiffness of concrete deck to global stiffness of composite floor

Using FEM modelling it is possible to observe influence of concrete deck transverse stiffness on global stiffness of composite floor. Of course this is possible only if concrete deck is also supported on sides parallel to timber beams. For a suitable proportion of floor sides and sufficient thickness of concrete layer a big difference between deck and
beam behaviour can be observed. To show this effect parametric analysis was prepared. On the floor with span of 6m in the direction of timber beams with cross section of 100/200mm, transverse span was varying from 3 to 12m. Supports were considered on the ends of timber beams and also on sides of concrete deck. Calculations were evaluated for standard thicknesses of concrete – 50mm, 60mm, 70mm and 80mm. Scheme is shown on figure 6. Results are presented by chart of deflection depending on proportion of sides of concrete deck on figure 7. Curves show deformation of floor for corresponding thickness of concrete layer. This deflection was calculated using full FEM model, i.e. whole concrete deck was modelled with shell elements supported on sides and connected to timber beams by interface as described in previous chapters. On the other hand, horizontal dashed lines show deflection calculated on 2D FEM model, i.e. only beam with effective cross section was modelled.

Fig.6: Scheme of parametric analysis

Fig.7: Results of parametric analysis

5. CONCLUSION
Paper deals with design of composite timber-concrete beams. Technical standard method and its modification is described. Second part of paper deals with modelling of these structures. A way, how to create FEM model of composite beam is described. From the comparison to classical calculation is obvious that this procedure is very accurate and can be used alternatively to standard method. In addition it also allows you to utilize reserves that are hidden in transverse stiffness of concrete deck. As showed on figure 7, for a suitable proportion of sides of concrete deck, these reserves can be big. On the other hand, for proportion of concrete sides higher than 1.5 this effect is negligible and it is better to consider only beam behaviour.

References


