INFLUENCE OF EXTERNAL LOADING ON FINAL SHAPE OF GRID SHELL STRUCTURE OBTAINED BY DYNAMIC RELAXATION METHOD

VPLYV VONKAJŠIEHO ZAŤAŽENIA NA VÝSLEDNÝ TVAR PRÚTOVEJ ŠKRUPINOVEJ KONŠTRUKCIE ZÍSKANÝ METÓDOU DYNAMICKEJ RELAXÁCIE

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Abstrakt

Hlavnou témou tohto článku je metóda dynamickej relaxácie (DRM) a jej aplikácia. Pomocou nej je možná optimalizácia tvaru konštrukcie, teda v tomto prípade prútovej škrupinovej konštrukcie. Za pomoci výpočtových nástrojov, ktoré dokážu spraviť za krátky čas množstvo prepočtov je možné analyzovať jednotlivé prípady so zohľadnením rôznych vstupných hodnôt. V metóde je niekoľko dôležitých vstupných hodnôt, ktoré významným spôsobom ovplyvňujú priebeh výpočtu a výslednú deformáciu konštrukcie. Článok je zameraný na externé zaťaženie aplikované v procese hľadania tvaru a jeho vplyv na výsledný tvar prútovej škrupiny. So zmenou typu a hodnoty zaťaženia dochádza ku zmene geometrie konštrukcie a teda aj výsledných vnútorných síl.

Kľúčové slová: *metóda dynamickej relaxácie, externé zaťaženie, prútová škrupinová konštrukcia*

Abstract

Dynamic relaxation method (DRM) and its application is the main topic of this paper. By DRM is possible to optimize the shape, in this case the shape of grid shell structure. With the aid of computational tools that provide us to make many calculations in a short time is possible to explore many problems considering different inputs. There are many important input values with significant impact on the course of calculation and the resultant displacement of structure. The paper is focused on external load applied in the process of form finding and its influence on final form of grid shell. By variation of type and value of loading there is a change of structure geometry and resultant internal forces.

Key words: *dynamic relaxation method, external loading, grid shell structure*

1 INTRODUCTION

Today's modern architecture heads to utilization of free shaped lightweight grid shell structures made of steel, wood and glass. To get the appropriate shape, stressed only by compression without effect of bending moments, it is necessary to deal with some form finding methods. In the past, there have been developed many methods to calculate the curved grid shell geometry according to equilibrium condition. Each method has various conditions to determine the stable state of each node of structure. The process of finding an equilibrium position is usually called as a stabilizing or form-finding process, whose result is highly efficient grid shell structure that can effectively resist against external load.

2 BASIC EQUATIONS OF DRM

DRM is a numerical optimization method for solving highly non-linear problems. The technique traces the motion of the structure through time under applied load [1]. It means that the method is tracking the movement of each structural node from initial unloaded position, during each time step, until the structure reaches the static equilibrium as a result of artificial damping. It means that a sum of all forces acting in the node is equal to zero. According to D' Alembert's Principle can be written equilibrium condition for the motion of system, which must be fulfilled at each time step, consider the inertial and damping forces [2] (1).

$$M^n \cdot \ddot{v}^n + C^n \cdot \dot{v}^n + K^n \cdot v^n = P^n \tag{1}$$

Where, M^n, C^n and K^n are fictitious mass, viscous damping and stiffness and P^n vector of external applied loads, $\ddot{v}^n$ and $\dot{v}^n$ are derivations of displacement vector v^n. Superscript *n* marks iterative step (fictitious time step). In this case we will utilize only kinetic damping that is used by reason of the rapid and stable convergence in the calculation. Therefore the viscous damping component in equation (1) can be ignored. If

$$R^n = M^n \cdot \ddot{v}^n \tag{2}$$

then by substitution (2) to (1) and by additional treatment is obtained the equation for the residual forces (3):

$$R^n = P^n - K^n \cdot v^n = P_i^n - \sum_{m=1}^{q} \frac{E_m \cdot A_m}{l_m^2}(x_j - x_i) \cdot v^n \tag{3}$$

Where E_m is the Young's modulus for element *m*, A_m is the cross section area and l_m the length of element *m* between nodes *i* and *j*. The movement of each node is caused by residual forces (3), that are the result of summation of internal ($K^n \cdot v^n$) and external forces (P^n) acting in the node. Equilibrium state is reached when residual forces are close to zero.

Kinetic energy (4) of whole structure is controlled during entire calculation,

$$E_k = \frac{1}{2} \cdot \sum_{i}^{n} \sum_{j}^{m} M_{ij} \cdot v_{ij}^2 \tag{4}$$

where M_{ij} are nodal masses, v_{ij} are velocities of node *i* in x, y and z axis. On the one hand, its value and on the other hand, its difference to the previous time step is checked. If the current value is smaller, energy peak occurs and the kinetic damping is applied. It means practically that all nodal velocities are set to zero and current coordinates are input values for next iteration. Iterative calculation is carried out until an energy peak, which is smaller than a chosen limit value is reached. The iteration process ends when all convergence conditions for out of balance force and kinetic energy (e_R=1.0E^{-6} and e_K=1.0E^{-12}) [2] are fulfilled.

3 INPUT VALUES IN DRM

The resulting point position depends on a multiple input parameters. The best way to show their effects is to change one of the parameters and keep all others fixed. Out of many components that can vary, several of the most important ones are chosen. Namely, the results of DRM optimization will be affected by the following parameters: fictitious mass, Young's modulus of elasticity, ultimate stress, type of damping, pattern of grid, cross-section, load and support combinations. Following chapters deal with the influence of external loading applied in the process of form-finding.

1.1 Form-finding of grid shell structure

The final shape was obtained by software based on the DRM terms, with which is possible to analyze single shapes with different ground plans, boundary conditions, loading. It also allows tracing the process of form finding and seeing the influence of input values on calculation and on final form of grid shell.
For numerical verification of kinetic DRM and effect of external loading some analyses have been made. The shaping process started with the initially flat rectangular ground plan with dimensions 10 m x 15 m. The boundary conditions were considered as pin joined at the edges in longitudinal direction. The mesh type was triangular grid with steel profiles of 80 mm diameter and 4 mm wall thickness.
The external load was applied before starting the process of DRM. Two cases were analyzed. The first geometry was found by form-finding process with symmetrical (undrifted) load. This load was applied into each free node of structure with value -2.5 kN (Fig. 1). Based on these initial conditions and the parameters inserted in Table I the cylindrical grid shell was obtained. Its final structure cross-section is symmetrical, which is the result of applied symmetrical loading. The final geometry is shown in the Fig.1.

Table. 1: The material parameters and constants for calculation

Parameter		
Young's modulus	E	1.0 GPa
Poisson ratio	ν	0.3
Cross-sectional area	A	9.55E-4 m^2
External load	P	symmetrical (Case I) / unsymmetrical (Case II)
Mass factor	k_m	0.1

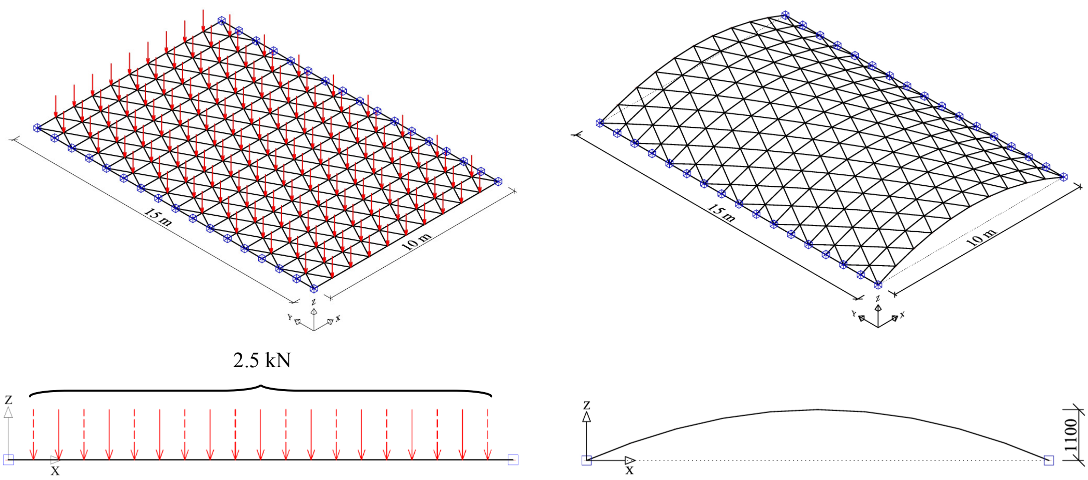

Fig. 1: The initial ground plan and the final shape (symmetrical external load)

The second analysis was focused on unsymmetrical load and its influence on final shape obtained by DRM. The scheme and arrangement of loading is in the Fig. 2. This “triangular” shape is representing the snow load on cylindrical roof. In the process of form-finding isn’t necessary to work exactly with the same values of loading as determined by the European standard. How was written before, loading is one of many input parameters affecting the resulting point position (x, y, z coordinate). Important in this case is the type (shape) of load. The final cross-section isn't symmetrical by reason of unsymmetrical load acting during form-finding process (Fig.2, right).

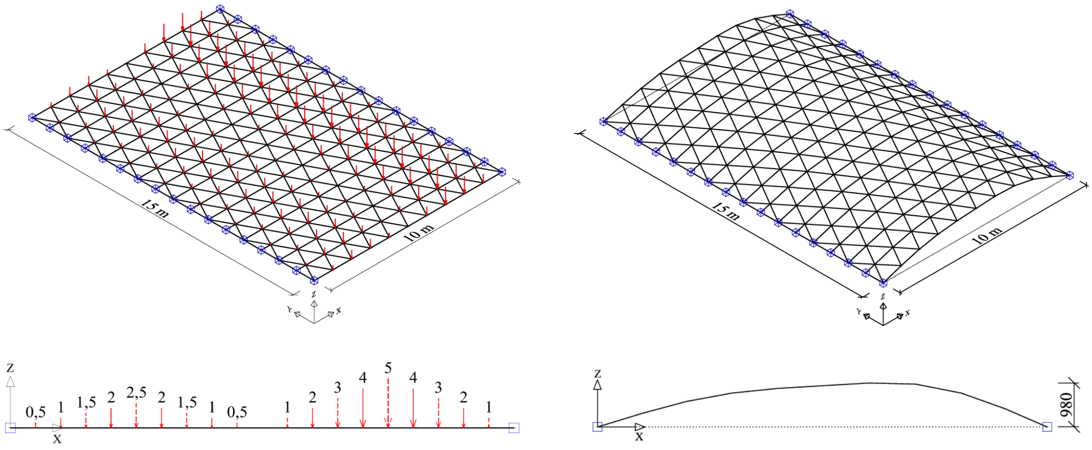

Fig. 2: The initial ground plan and the final shape (unsymmetrical external load)

1.2 Estimation of external load on cylindrical roofs

The snow load on cylindrical roof was estimated according to EN 1991-1-3 [4]. The undrifted and drifted load arrangement was taken into account (Fig. 3). The characteristic value of snow load on the ground for a given location was specified according to STN EN 1991-1-3/NA 1 [5]. The roof shape coefficient was estimated on the basis of final geometry from DRM. The thermal coefficient C_t was taken with reduced value on the ground of high thermal transmittance of glass covered roof. According to this inputs the snow load on roof for persistent/transient and also accidental design situation was determined with values shown in Fig. 3.

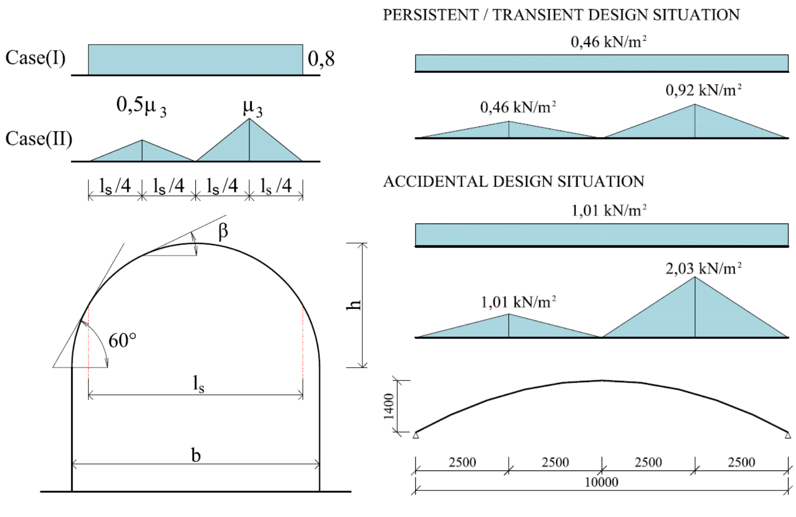

Fig. 3: Snow load shape coefficients and the resultant snow load for cylindrical roof

1.3 Structural analysis by FEM modeling

The final geometries from DRM were loaded into the software working according to FEM. The grid profiles were modeled using steel beam elements with the same cross-sectional properties like in the DRM analysis (tube section 80x4). Between the profiles were modeled rigid connections. In the both sides of structure were used pinned boundary conditions. In the analysis was considered with snow load. Its values were calculated for the cylindrical structure with 10m span and 1,1m high of arch. Only the accidental design situation was taken into the account, because of higher values and consequently less favorable action of load. Two load cases were created, the first for symmetrical and the second for unsymmetrical snow load action. Both cross-sections of structures and its loadings are shown on Fig.4 and Fig.5. Under these conditions the calculation was carried out.

From resulting axial forces and bending moments is obvious effect of geometry of the structure on values of internal forces. From internal forces results the influence of unsymmetrical load which significantly increase the values of bending moments. It has negative impact to resistance of beams and global deformations in this type of structures.

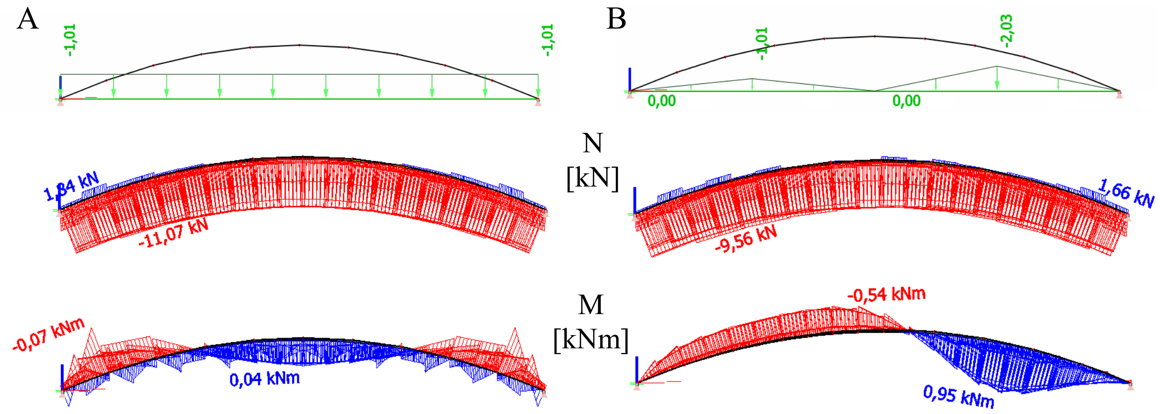

Fig. 4: The symmetrical geometry of cylindrical shell and variation of internal forces, a) symmetrical load, b) unsymmetrical load

On the other hand the second geometry was obtained from unsymmetrical „triangular“ load. Therefore this structure is better adapted to carry this type of loading. As you can see in Fig. 5 B, the values of bending moments decreased significantly, which has a positive impact in terms of the normal stresses. By contrast, from symmetrical load, the values of axial forces and bendig moments are higher (Fig. 5 A) what is unfavourable for such structure.

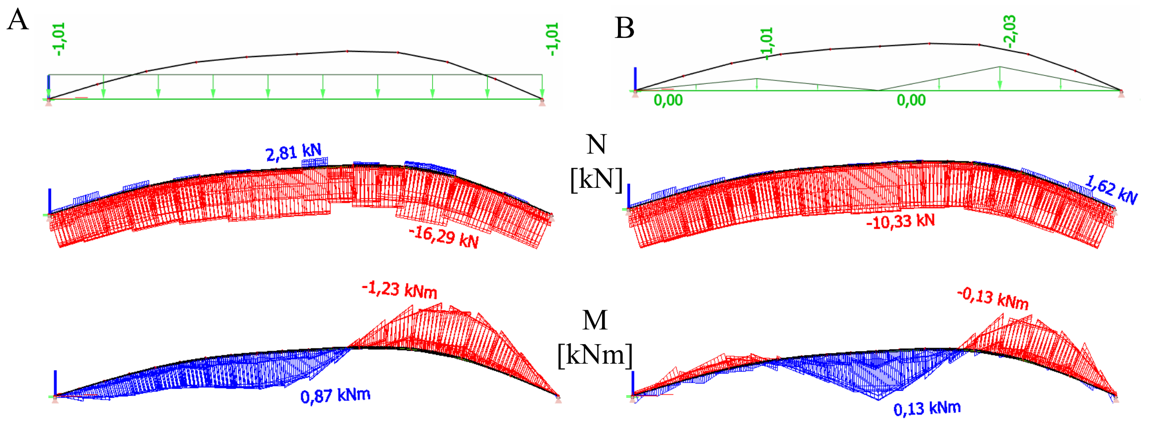

Fig. 5: The unsymmetrical geometry of cylindrical shell and variation of internal forces, a) symmetrical load, b) unsymmetrical load

1.4 Conclusion

The final grid shell geometry can be obtained by form finding process, where is the displacement of nodes significantly affected by single input parameters. Currently, there are some methods, which are concerned with optimizing the shape. In this article are mentioned basic equations of DRM and described the impact of loading type on final geometry of structure. The analysis was focused on the influence of load on cylindrical shell. The shapes were obtained by DRM from symmetrical and unsymmetrical load. These two final geometries were analyzed by FEM. The results show that the geometry obtained from form finding process for one load case is effective only for that specific case. Therefore, it would be appropriate to find the optimal shape of the structure based on the combination of several load cases, or the envelope.

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